

TABLEAUX 2009 WORKSHOP
ON
GENTZEN SYSTEMS AND BEYOND

BOOKLET OF ABSTRACTS

EDITORS
KAI BRÜNNLER AND GEORGE METCALFE

JULY 6, 2009

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A SIMPLE SEMANTIC PROOF OF COMPLETENESS AND CUT-ADMISSIBILITY
FOR AN HYPERSEQUENTIAL CALCULUS FOR PROPOSITIONAL GÖDEL LOGIC

Arnon Avron

Tel-Aviv University

aa@tau.ac.il

We provide a constructive, direct, and simple semantic proof of the completeness of the cut-free part of the hypersequential calculus for Gödel logic (thereby proving both completeness of the calculus for its standard semantics, and the admissibility of the cut rule in the full calculus). We then extend the results and proofs to derivations from assumptions, showing that such derivations can be confined to those in which cuts are made only on formulas which occur in the assumptions.

Marta Bílková

Charles University, Prague

`marta.bilkova@ff.cuni.cz`

The uniform interpolation property for a propositional logic states that for every formula ϕ and any choice of its propositional variables \bar{q} , there is a post-interpolant (a right interpolant) $E_q(\phi)$ such that for all ψ , whenever $(\phi \rightarrow \psi)$ is provable and the shared variables of ϕ and ψ are among \bar{q} , $(\phi \rightarrow E_q(\phi))$ and $E_q(\phi) \rightarrow \psi$ are provable. Similarly a pre-interpolant (a left interpolant) can be defined. The right (left) uniform interpolants can be seen as formulas which describe, forgetting specified variables, which part of a formula is responsible for entailing (being entailed by) formulas sharing a specified sublanguage of ϕ . They often coincide with bisimulation propositional quantifiers. Such quantifiers quantify over possible values of a proposition not just in a fixed model but in any model bisimilar to it. Rather rarely they can be expressed in the propositional language itself. If so, they act as uniform interpolants, see [5, 10]. Proofs of a uniform interpolation theorem based on this connection can be found in [10] (a purely semantic approach), we will however concentrate on proofs based on a proof theoretical argument [7, 2, 1].

A more elegant proof of uniform interpolation can sometimes be given - uniform interpolants can be constructed simply removing occurrences of specified atoms from a normal form of a given formula. Such a proof for the modal logic K can be found in [8]. A suitable modal normal form arises in connection to a coalgebraic view on modal logic (this approach originated in [6], for an overview see [9]). The proof theoretical proofs of uniform interpolation theorems mentioned above can be seen as quite close to the situation where we simply remove occurrences of an atom during steps of a proof search procedure, which indeed can be seen as constructing a normal form. One can hope to visualize construction of such normal forms by a suitable proof system. As an example we consider two systems for basic modal logic K given in a language with a coalgebraic cover modality taken as primitive [3]. We will check a possibility of modal deep inference calculi introduced in [4] to visualize uniform interpolation proofs. For this to work we propose another deep inference system for the basic modal logic K given in the ordinary language more suitable for this purpose. The system uses the pattern of the cover modality on the level of structure of sequents.

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Nikolaos Galatos

University of Denver

ngalatos@du.edu

Joint research with **Agata Ciabattoni** and **Kazushige Terui**

Proof theory has proved very useful in analyzing and studying the properties of logical systems. In particular, most systems that are amenable to proof theoretic analysis enjoy the cut-elimination property. The proof of this property, however, usually requires tedious verification of various cases and has to be performed again every time modifications are made to the original system. Focusing on propositional substructural logics (and in particular on the system FL), we will present a short algebraic proof of cut elimination, based on the notion of residuated frames. The proof covers extensions of FL with analytic structural rules in a modular way, and also extends to hypersequent calculi.

The algebraic models of the logic corresponding to the system FL are (pointed) residuated lattices. Residuated frames, on the other hand, are semantic structures that encompass both residuated lattices, but also proof-theoretic aspects of FL. The study of residuated lattices and frames, in conjunction with ideas from linear logic and the notion of polarity, has indicated a classification of propositional formulas in a hierarchy (P_n, N_n) , which we call the substructural/formula hierarchy. Formulas at the N_2 level of the hierarchy correspond to the structure of the system FL and are equivalent to structural rules of FL. Likewise, P_3 formulas correspond to the hypersequent system HFL and to structural rules over that system. The hierarchy actually indicates the form of systems of higher complexity, whose structural rules can capture more complicated formulas of the hierarchy.

We describe the exact correspondence between formulas and (hyper)structural rules and investigate syntactic criteria that guarantee that the equation is equivalent to an analytic structural rule. Moreover, we provide an algorithm for transforming equations into the corresponding rules. Finally, we outline the connections with algebraic (Dedekind-MacNeille) completions and with conservativity of the infinitary logic over FL.

TOWARDS AUTOMATIC CUT ELIMINATION

Rajeev Goré

Australian National University, Canberra

Rajeev.Gore@rsise.anu.edu.au

Joint research with **Jeremy Dawson** and **Revantha Ramanayake**

Modern proof-assistants are now mature enough to formalise many aspects of mathematics. I outline some work we have done using the proof-assistant Isabelle to machine-check aspects of proof theory in general, and specifically the cut-elimination theorem for provability logic GL which has a contentious track record.

BEYOND

Alessio Guglielmi

INRIA Nancy-Grand Est and University of Bath

After almost ten years of experience with deep inference, we draw some conclusions about the adoption of locality in structural proof theory.

A deductive system adopting the principle of locality is such that verifying the correctness of each inference step can be done in constant time. We can consider locality a generalisation of linearity, and it is one of the most peculiar properties of deep inference. In fact, we can only have access to locality if we go beyond Gentzen proof theory, and we adopt more general formalisms than traditional ones.

As expected, locality is favouring the emergence of geometrical methods in the design and normalisation of deductive systems. However, we are also seeing unexpected and somewhat weird phenomena, like quasipolynomial cut-elimination for propositional logic.

I will give an idea of these aspects, and then I will illustrate current, related research on the problem of proof identity.

THE COALGEBRAIC MU-CALCULUS

Dirk Pattinson

Imperial College London

`dirk@doc.ic.ac.uk`

Joint research with **Clemens Kupke** and **Corina Cirstea**

Applications of modal logics are abundant in computer science, and a large number of structurally different modal logics have been successfully employed in a diverse spectrum of application contexts: knowledge representation, reasoning about distributed and multi-agent systems and the verification of distributed systems. Coalgebraic semantics provides a uniform and encompassing view on the large variety of specific logics used in particular domains. The coalgebraic approach is generic and compositional: tools and techniques simultaneously apply to a large class of application areas and can moreover be combined in a modular way.

We summarise the coalgebraic framework, along with some recent developments, and then introduce the coalgebraic mu-calculus, an extension of the general (coalgebraic) framework with fixpoint operators. Our main results are completeness of the associated tableau calculus and EXPTIME decidability. Technically, this is achieved by reducing satisfiability to the existence of non-wellfounded tableaux, which is in turn equivalent to the existence of winning strategies in parity games. Our results are parametric in the underlying class of models and yield, as concrete applications, previously unknown complexity bounds for the probabilistic mu-calculus, and for an extension of coalition logic with fixpoints.

ON THE CORRESPONDENCE BETWEEN HYPERSEQUENT
AND LABELLED CALCULI FOR INTERMEDIATE LOGICS

Robert Rothenberg

University of St. Andrews

rr@cs.st-andrews.ac.uk

Hypersequent calculi were first introduced independently by Beth (1959), Pottinger (1983) and Avron (1987) as an extension of sequent calculi suitable for some non-classical logics. This formalism allows for the development of *analytic* calculi for a wide-range of logics—that is, cut-free calculi with the same logical symbols as the logics they are intended to be proof-systems for.

Labelled sequent calculi, apparently first introduced by Kanger (1957), are another extension of sequent calculi where formulae are annotated with labels which correspond with some *semantic* interpretation, and the sequents may include other kinds of formulae that indicate the relationship between labels (e.g., accessibility relations in a Kripke frame).

Our work shows that these formalisms are essentially the same for *intermediate logics* (logics between **Int** and **Class**): we can think of labels as names for components, and relational formulae as indicating how components are related in terms of proof search. This seems to be a part of the folklore of hypersequents, but aside from the relationship between formalisms for the modal logic **S5**, we are unaware of other work which discusses this as a *general* correspondence.

To show this correspondence, we first show a bijection between hypersequents and labelled sequents without relational formulae. We then extend this to labelled sequents with relational formulae by using an intermediate translation to a fragment of first-order logic, and show that the general semantics are equivalent, i.e.

$$\bigvee_{i=1}^n (A_i \supset B_i) \equiv \left(\bigwedge_{i=1}^n A_i \right) \supset \left(\bigvee_{i=1}^n B_i \right)$$

We also show, using proof-theoretic as well as semantic (via the translation to first-order logic) methods, that rules which introduce relational formula and “fold” sequents, e.g.:

$$\frac{x \leq y, \Sigma, \Gamma^x, \Gamma^y, \underline{\Gamma}' \Rightarrow \underline{\Delta}}{x \leq y, \Sigma, \Gamma^x, \underline{\Gamma}' \Rightarrow \underline{\Delta}} L \leq \quad \frac{x \leq y, \Sigma, \underline{\Gamma} \Rightarrow \underline{\Delta}', \Delta^x, \Delta^y}{x \leq y, \Sigma, \underline{\Gamma} \Rightarrow \underline{\Delta}', \Delta^y} R \leq$$

are admissible.

What makes this correspondence of interest for automated reasoning is that it allows one to separate the underlying formalism used in an implementation from the formalism that the user works with.

CUT-FREE NESTED SEQUENT CALCULI FOR LOGICS WITH ADJOINT PAIRS
OF MODALITIES

Mehrnoosh Sadrzadeh and Roy Dyckhoff

Oxford University; St Andrew's University

mehrs@comlab.ox.ac.uk, rd@st-andrews.ac.uk

Among the many formalisms following those introduced by Gentzen for capturing the provability of formulae in non-classical logics, we are interested in those variously called something like deep sequents or nested sequents. In our treatment, these have some kind of structure as an antecedent and a single formula as the succedent. The structures, called by us contexts, are defined by mutual recursion with a class of items: formulae are *items*, annotated contexts are *items* and multisets of items are *contexts*. The closest match in the literature appears to be in the work of Moortgat [4], who attributes them to Belnap. Other related ideas are attributed to (e.g.) Kashima [3].

Inference rules then involve the analysis of formulae both on the left and on the right, with the analysis on the left being sometimes only allowed on the surface and at other times being allowed deep inside the context. A careful treatment of the operations on contexts is therefore required. Standard structural rules such as *Weakening*, *Contraction* and *Cut* can then, provided the rules have been chosen carefully, be proved admissible.

Such calculi appear to be suitable for formalising logics with adjoint pairs (F, G) of modal operators, such as the pair (\diamond^{-1}, \square) for “sometime in the past” and “always in the future from tense logic, especially when the logic is positive in the sense of Dunn [2] (i.e. with no implication or negation). By adjoint we mean just that, for arbitrary formulae m, n , $Fm \vdash n$ iff $m \vdash Gn$; provided F and G are monotone, this is enough to imply that, for all m , $M \vdash GFm$ and $FGm \vdash m$. (interpreted e.g. as “if m is true (now) then always in the future it will be the case that m was true sometime in the past”).

Outside the field of tense logic, such logics arise in the first authors treatment (with two others) of information (and mis-information) flow [1]; the paper [5] gives a formal treatment of an appropriate generalised sequent calculus, with contexts and items as above, for dealing with a fragment of such a logic. We propose to summarise this, indicating the issues in proving admissibility and thus completeness results; if time permits, we will also give an application to tense logic, dealing with the problems raised by the relation between “sometime in the past” and “always in the past” when negation is absent

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EXPANDING THE REALM OF SYSTEMATIC PROOF THEORY

Lutz Strassburger

École Polytechnique, Laboratoire d'Informatique (LIX)

lutz@lix.polytechnique.fr

Joint research with **Agata Ciabattoni** and **Kazushige Terui**

This paper is part of a general project of developing a systematic and algebraic proof theory for nonclassical logics. Generalizing our previous work on intuitionistic-substructural axioms and single-conclusion (hyper)sequent calculi, we define a hierarchy on Hilbert axioms in the language of classical linear logic without exponentials. We then give a systematic procedure to transform axioms up to the level P'_3 of the hierarchy into inference rules in multiple-conclusion (hyper)sequent calculi, which enjoy cut-elimination under a certain condition. This allows a systematic treatment of logics which cannot be dealt with in the previous approach. Our method also works as a heuristic principle for finding appropriate rules for axioms located at levels higher than P'_3 . The case study of Abelian and Lukasiewicz logic is outlined.