A VALUATION-BASED ARCHITECTURE FOR ASSUMPTION-BASED REASONING*

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ABSTRACT†

Starting from assumption-based propositional knowledge bases, a symbolic evidence theory[3] can be developed. This theory is the qualitative equivalent of the well known numerical Dempster-Shafer theory of evidence[5]. Given a propositional knowledge base and some additional facts or observations, the problem is to compute the quasi-supports, the contradictions and the supports in a symbolic form for one or a few hypotheses. The main advantage of this approach is that symbolic supports are pure arguments in favour or against certain hypotheses and they can be transformed into linguistic explanations.

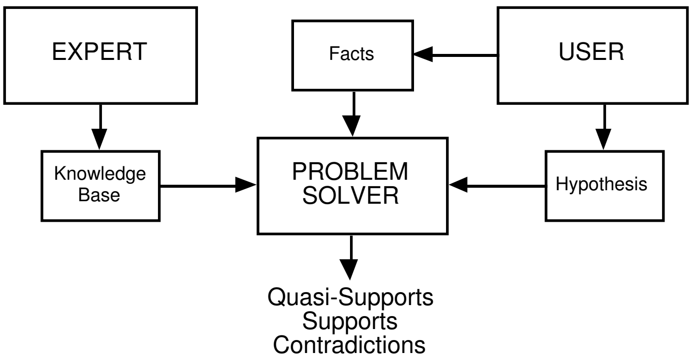

Figure 1. Given a propositional knowledge base and some additional facts or observations, the problem is to compute the quasi-supports, the contradictions and the supports.

*Research supported by grants No. 21-30186.90 and 21-32660.91 of the Swiss National Foundation for Research, Esprit Basic Research Activity Project DRUMSII (Defeasible Reasoning and Uncertainty Managment).

†This is a short summary of the actual state of a research work which will be published later as a part of the author's Ph.D. thesis. The mathematical foundations and most of the theoretical background are published by Kohlas[3].

There exist different ways to treat symbolic evidence theory. One approach is to compute the set of all prime implicates and to apply the filtering theorems of Reiter and DeKleer[4]. This approach is only reasonable for very small examples. The computation of all prime implicates leads to serious problems of complexity. A better method, the so-called SOL-resolution (skipped ordered linear resolution), was developed by Siegel[7] and is implemented in Prolog III[1]. It works even for large examples but it is mainly useful if only relatively few queries are to be treated[2].

Alternatively, the symbolic theory of evidence fits into the axiomatic framework of valuation networks (Shenoy[6]). This leads to a local combination scheme for propagating symbolic arguments and supports similar to the methods of propagating probabilities or belief function. The benefit of this approach results from the fact that combinations can always be computed locally. The aim of this work was to develop an efficient valuation-based system which computes symbolic evidences. The system's architecture is based on the modules for the following seven tasks:

1. Transformation of the user's model description given in an appropriate modeling language into a set Σ of clauses.
2. Decomposition of Σ into disjoint sets of clauses Σ_i with $\cup \Sigma_i = \Sigma$. The sets P_i of propositional symbols contained in Σ_i are the hyperedges of a hypergraph.
3. Computation of all non-empty basic arguments for each set Σ_i. Basic arguments are a more efficient representation of supports containing less redundancies. A set of basic arguments is a valuation in the sense of Shenoy's framework of valuation networks[6] and it is called a symbolic hint.
4. Transformation of the hypergraph formed by the set of symbolic hints into a valuation network.
5. Propagation of the symbolic hints through the valuation network and computation of the marginals.
6. Production and display of the supports of the hypotheses in a minimal symbolic form.
7. Possibly transformation of the symbolic supports into numerical degrees of support.

The interesting point of this method is the decomposition of the knowledge base at the beginning of the compilation process. This divides the whole problem into different pieces which can be considered independently. It is no longer necessary to work always on the whole knowledge base Σ, i.e. on the whole set of clauses. This can reduce the complexity of the compilation process considerably.

There exist many different ways to decompose the knowledge base Σ. One approach is to consider each clause of Σ as a node of the hypergraph. In most applications this leads to very complex hypergraph structures with many cycles and it requires a fast algorithm to find a covering hypertree. Another approach is to build disjunct sets of assumptions and to distribute the clauses over these sets. In many applications this method takes into account the natural structure of the knowledge base and it leads to reasonable hypergraph structures.

After the decomposition of Σ each set of clauses Σ_i is compiled independently. If such a Σ_i contains a set P_i of n_i different propositions, the compilation of Σ_i means the computation of the basic arguments for all of the $2^{2^{n_i}}$ formulas in $\mathcal{L}_{P_i}$. A basic argument for a hypothesis h is a conjunction of literals over the assumptions which permits to prove h, but nothing more precise than h. There may be several basic arguments for a hypothesis. Basic arguments are the equivalent of the basic probability assignments in the numerical theory of evidence. For most of the $2^{2^{n_i}}$ different hypotheses the set of basic arguments is empty and doesn't need to be stored. Generally there are only a few non-empty sets of basic arguments. Let's also mention that supports can be computed from the basic arguments and inversely.

The transformation of a hypergraph structure into an optimal covering hypertree is a well-known NP-complete problem, but there exist several heuristics to find practicable hypertrees. In many applications the Maximum Cardinality Search (MCS) by Tarjan and Yannakakis[8] leads to satisfactory results. After the covering hypertree has been computed the next step is the construction of the valuation network which is a very simple task. Then the propagation process can be started, i.e. the symbolic hints are locally combined through the whole network and the marginals of the nodes are computed. The complexity of the propagation process depends on the maximal number of propositions contained in the nodes of the tree.

The marginals computed at each node of the network can be used to answer the queries. A query is a logical formula over the set of propositional symbols of a node. The result of a query is a logical representation of the support of the query. As already mentioned above, the supports can easily be derived from the basic arguments contained in the marginals of the nodes. The resulting logical form tells whether the assumptions have to be true or false to prove the hypothesis. In many cases some assumptions are irrelevant to the provability of certain hypotheses. Having the supports in a minimal logical (i.e. symbolic) form it is possible to derive the corresponding numerical degrees of support. Techniques for this computation can be found in the domain of reliability theory where similar problems have to be solved.

The advantage of combining and propagating symbolic arguments rather than probability assignments resides in two points. First, obtaining symbolic arguments may help to explain why a hypothesis is credible or not, because the arguments for and against a hypothesis are explicitly given. Secondly, once the symbolic supports are known, it is easy to perform sensitivity analysis on the probabilities assigned to the assumptions and to analyze the relative importance of the different assumptions for the judgment of the hypothesis. The disadvantage of this approach is that computations are much more expensive than pure probability propagation.

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